7-1 Integral as Net Change

Learning Targets

Given a differential equation, I can use integration to find a net change in a real world situation.

Given a differential equation and a starting value, I can use integration to find a the ending value in a real world situation.

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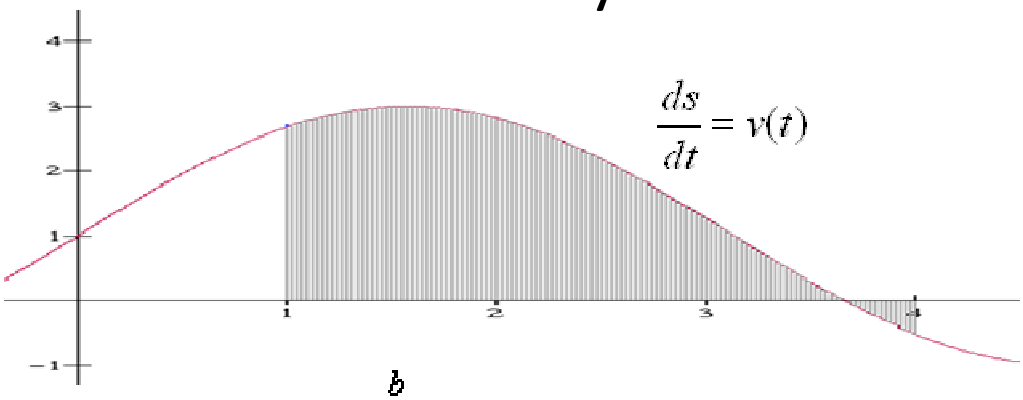
Velocity



- What does $\int_a^b v(t)\,dt$ mean?

- If s(a)=12 and $\int_a^b v(t)\,dt = 9$, what is s(b)?

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Integral as a Net Change

If f is a continuous and differentiable function over [a,b], then

$$f(b) = f(a) + \int_a^b f'(x)dx$$

And the integral $\int_a^b f'(x)dx$ tells you how much the function has changed from a to b.

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Example 1: $\frac{dp}{dt} = v(t) = t^2 - \frac{8}{(t+1)^2} \frac{cm}{\sec}$

is a velocity function on $0 \le t \le 5$

- Graph the velocity for
- Describe the motion.
- What is the particle's position at time t=1 sec and at t=5 sec if p(0)=9?
- What was the total distance travelled from t=0 to t=5?

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NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

$\backslash Y_1 = X^2 - 8/(X+1)^2$

$\backslash Y_2 =$
$\backslash Y_3 =$
$\backslash Y_4 =$
$\backslash Y_5 =$
$\backslash Y_6 =$
$\backslash Y_7 =$

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ZOOM MEMORY
2↑Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig
8:ZInteger
9:ZoomStat
0↓ZoomFit

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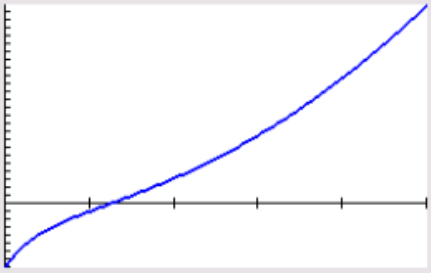
WINDOW
Xmin=0
Xmax=5
Xscl=1
Ymin=-8
Ymax=24.77777778
Yscl=1
Xres=1
ΔX=.01893939393939
TraceStep=.03787878787878

NORMAL FLOAT AUTO REAL RADIAN MP



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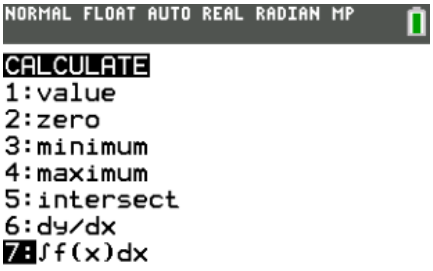
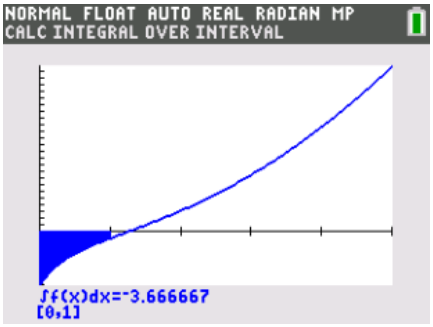
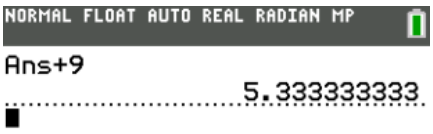
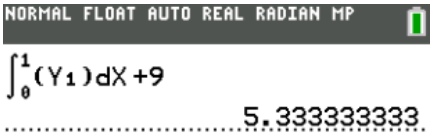
$$\int_0^1 v(t)\,dt + 9 \approx 5\tfrac{1}{3}\text{ cm}$$

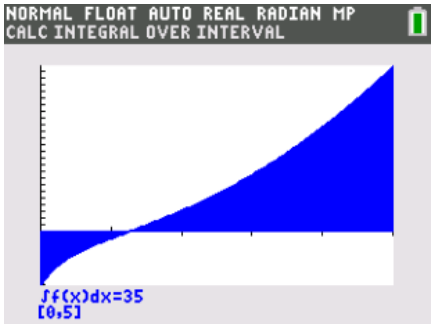


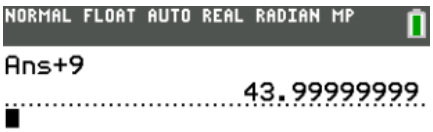


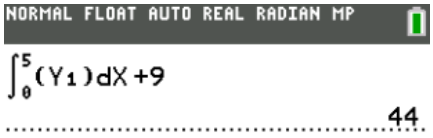




$$\int_0^5 v(t)\,dt + 9 \approx 44\text{ cm}$$







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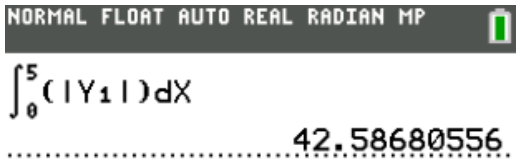
$$\int_0^5 |v(t)|\,dt$$

$$\approx 42.587 \text{ cm}$$



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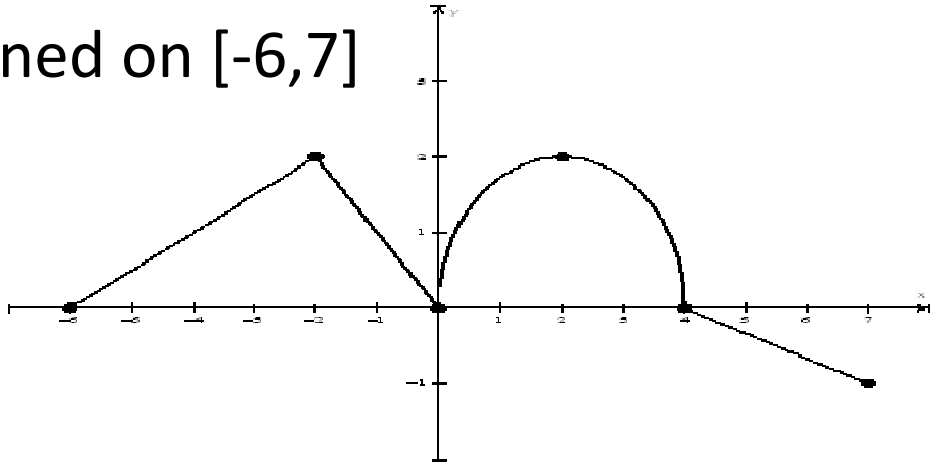
Example 2:

f'(x) is defined on [-6,7]

f(0)=5



- Find the absolute maximum of f(x) on [-6,7].
- Find the absolute minimum of f(x) on [-6,7].

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a.) Find the Absolute Max of $f(x)$ on $[-6,7]$.

candidates: ~~-6~~, ~~0~~, 4, ~~7~~

$x=-6$ & $x=7$ mins

$x=0$ inf pt b/c $f'(x)$ dresn't change signs

so $x=4$ is Abs Max

$$f(4) = 5 + \int_0^4 f'(x)\,dx$$

$$= 5 + \tfrac{1}{2}\pi r^2$$

$$= 5 + \tfrac{1}{2}\pi (2)^2$$

$$= 5 + 2\pi$$

Abs Max $= 5+2\pi$ @ $x=4$

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b.) Find the Abs Min of $f(x)$ on $[-6,7]$

Candidates: $x = -6, 0, 4, 7$

$x = 4$ Max
$x = 0$ inf pts

So either $x = -6, 7$

$$f(-6) = 5 + \int_{0}^{-6} f'(x)\,dx$$

$$= 5 + -\int_{-6}^{0} f'(x)\,dx$$

$$= 5 + -6 = -1$$

$$f(7) = 5 + \int_{0}^{7} f'(x)\,dx$$

$$= 5 + 2\pi + -\tfrac{1}{2} b \cdot h$$
$$= 5 + 2\pi + -\tfrac{1}{2} \cdot 3 \cdot 1$$
$$= 5 + 2\pi + -\tfrac{3}{2}$$
$$= 3\tfrac{1}{2} + 2\pi$$

Abs Min $= -1$ @ $x = -6$

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Homework

p. 386 #10-16, 19-22, 31-36

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